Rigid Plate Transformations to Support PPP and Absolute Positioning in Africa

Richard Stanaway & Craig Roberts

School of Surveying and Spatial Information Systems

University of New South Wales, Australia

FIG Working Week, Marrakech, Morocco, 18-22 May 2011

CORS Distribution in Africa

Sparse GNSS CORS infrastructure overcome by use of PPP and Global Differential Services

image: Centro GNSS de Canarias
www.canarygnsscenter.org

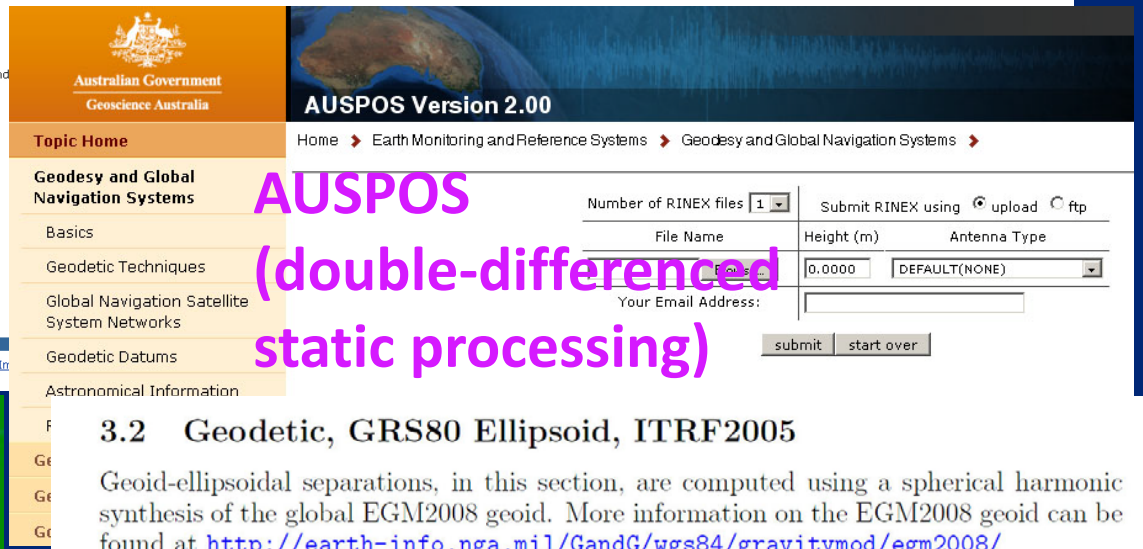
NRCan (PPP)

~~~~~~ rabt0010 ~~~~~~~~~~~~~~~~~~~~

The estimated coordinates / standard deviations for the rabt0010 RINEX file are as follow:

Latitude (ITRF05): 33 59 53.1751 (dms) / 0.003 (m)
Longitude (ITRF05): -6 51 15.4384 (dms) / 0.009 (m)
Ellipsoidal Height (ITRF05): 90.110 (m) / 0.019 (m)

UTM (North) Northing: 3764021.294m Easting: 698173.709m Zone: 29 Scale Factor: 1.00008



AUSPOS (double-differenced static processing)

OmniSTAR (RTK)

3.2 Geodetic, GRS80 Ellipsoid, ITRF2005

Geoid-ellipsoidal separations, in this section, are computed using a spherical harmonic synthesis of the global EGM2008 geoid. More information on the EGM2008 geoid can be found at http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/

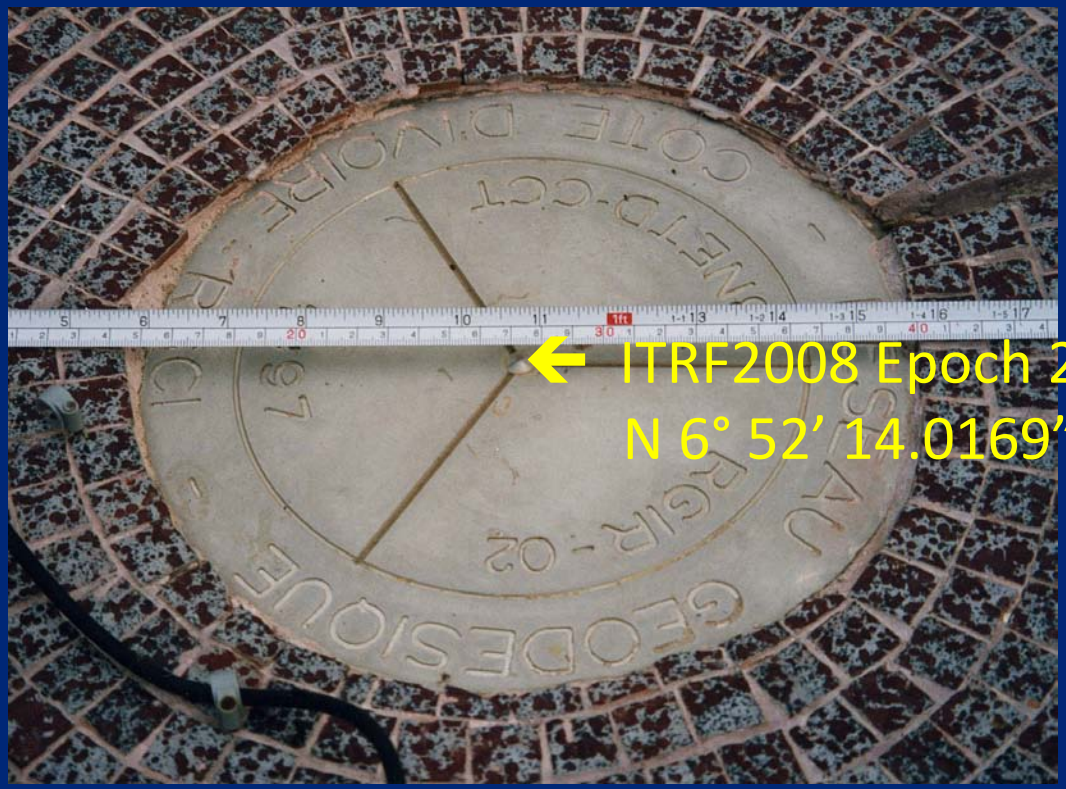
Station	Latitude (DMS)	Longitude (DMS)	Ellipsoidal Height (m)	Derived Above Geoid Height (m)
RABT	33 59 53.17590	-6 51 15.43821	90.153	44.805

4.1 Coordinate Precision - Geodetic, One Sigma

Station	σ East (m)	σ North (m)	σ Up (m)
RABT	0.001	0.002	0.003

ITRF positioning services (examples)
~~~~~~

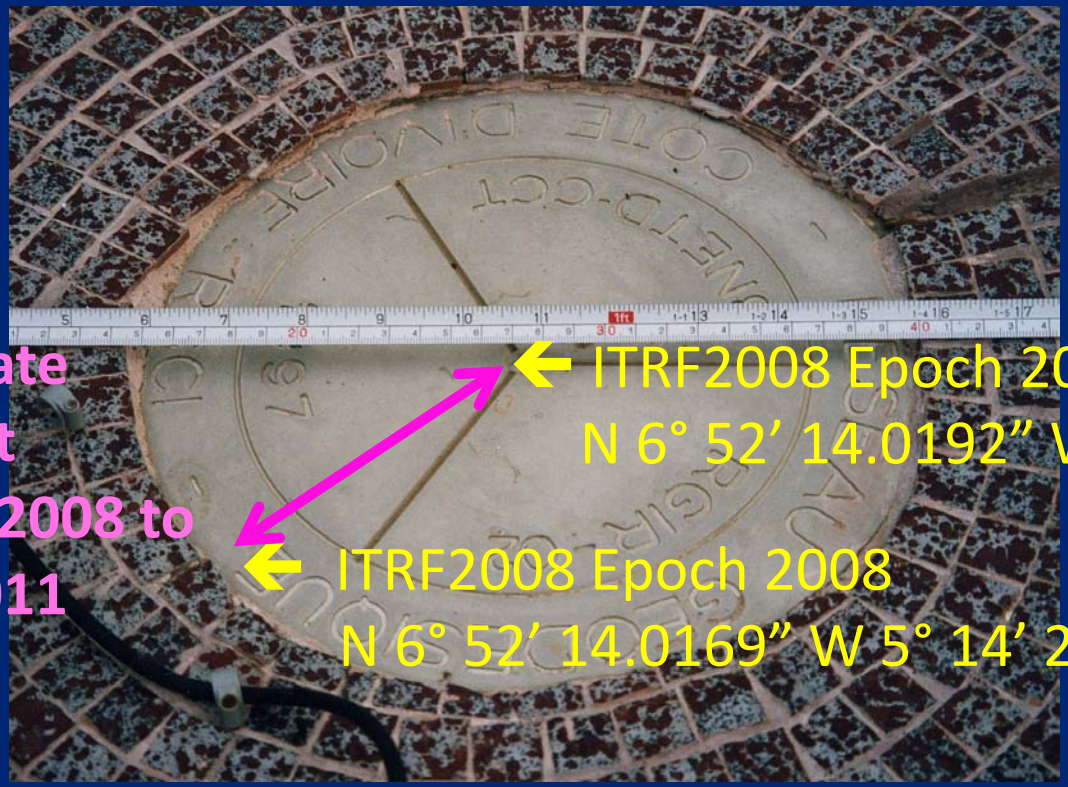
Kinematic coordinates illustrated



← ITRF2008 Epoch 2008
N 6° 52’ 14.0169” W 5° 14’ 24.3345”

YKRO – IGS Station
(Yamoussoukro, Cote d’ Ivorie)
From IGS web-site

Kinematic coordinates illustrated

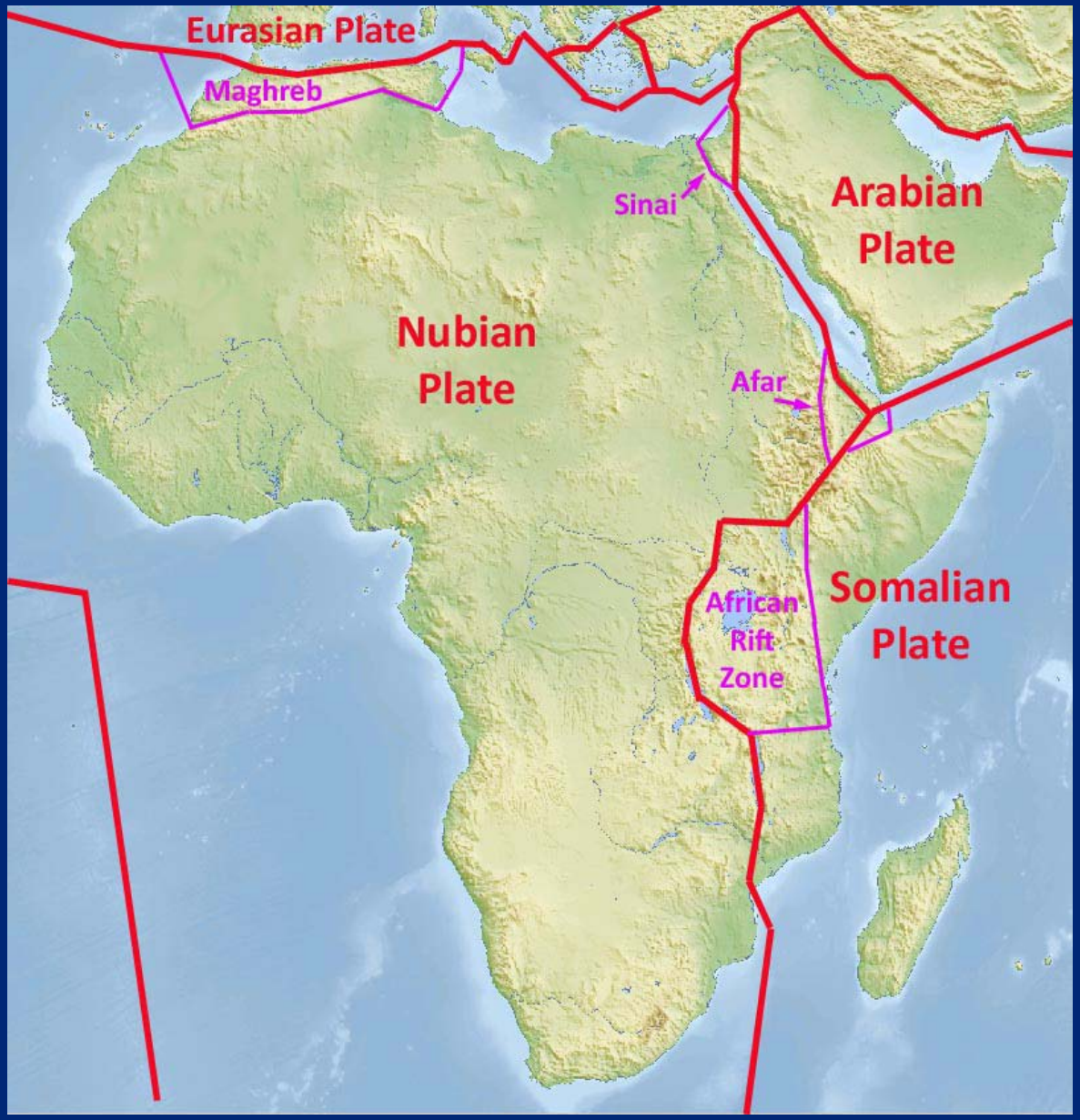


Nubian Plate movement 1 January 2008 to 22 May 2011 109 mm!

ITRF2008 Epoch 2011.4
N 6° 52’ 14.0192” W 5° 14’ 24.3317”

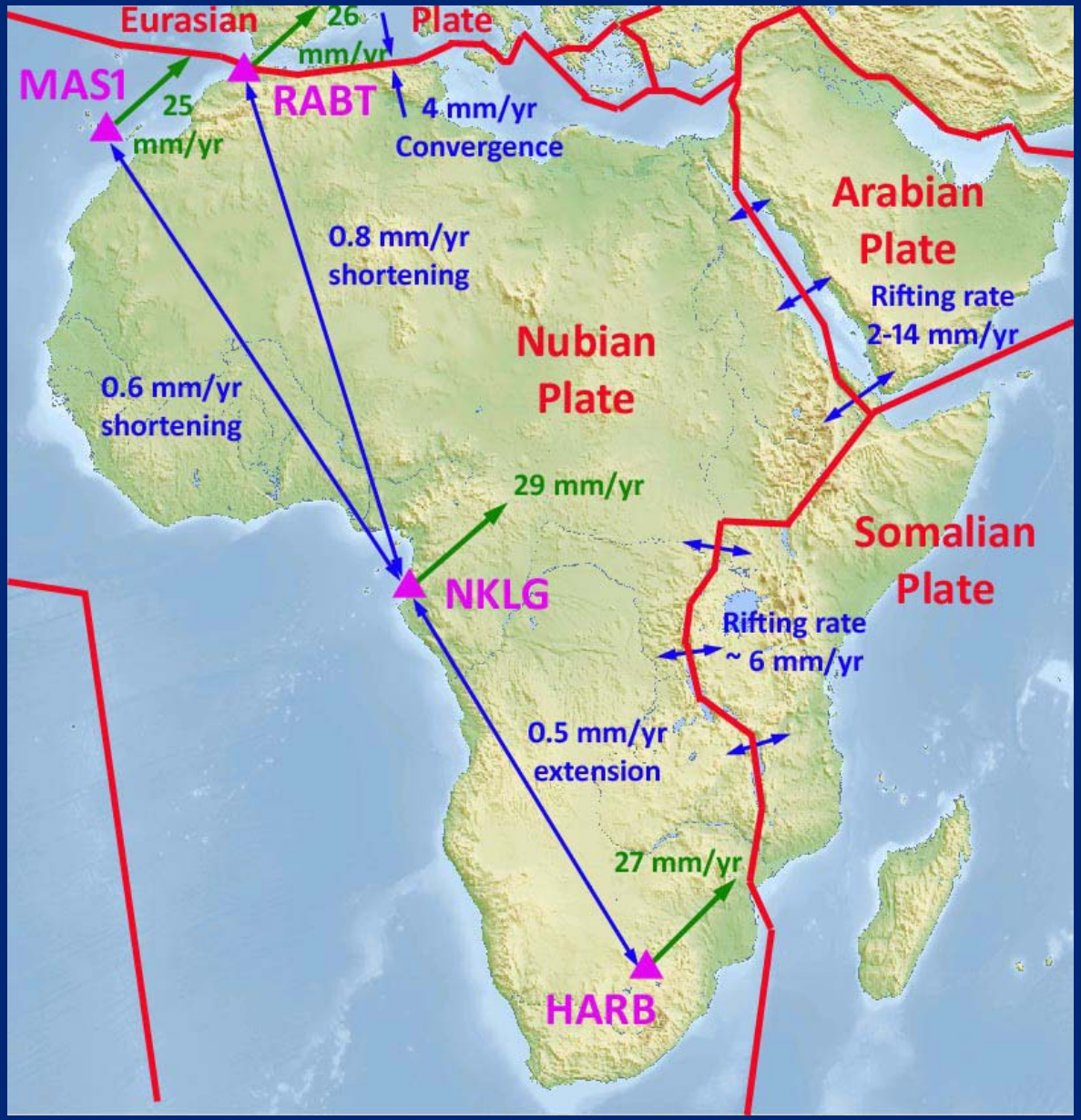
ITRF2008 Epoch 2008
N 6° 52’ 14.0169” W 5° 14’ 24.3345”

YKRO – IGS Station
(Yamoussoukro, Cote d’ Ivorie)
From IGS web-site



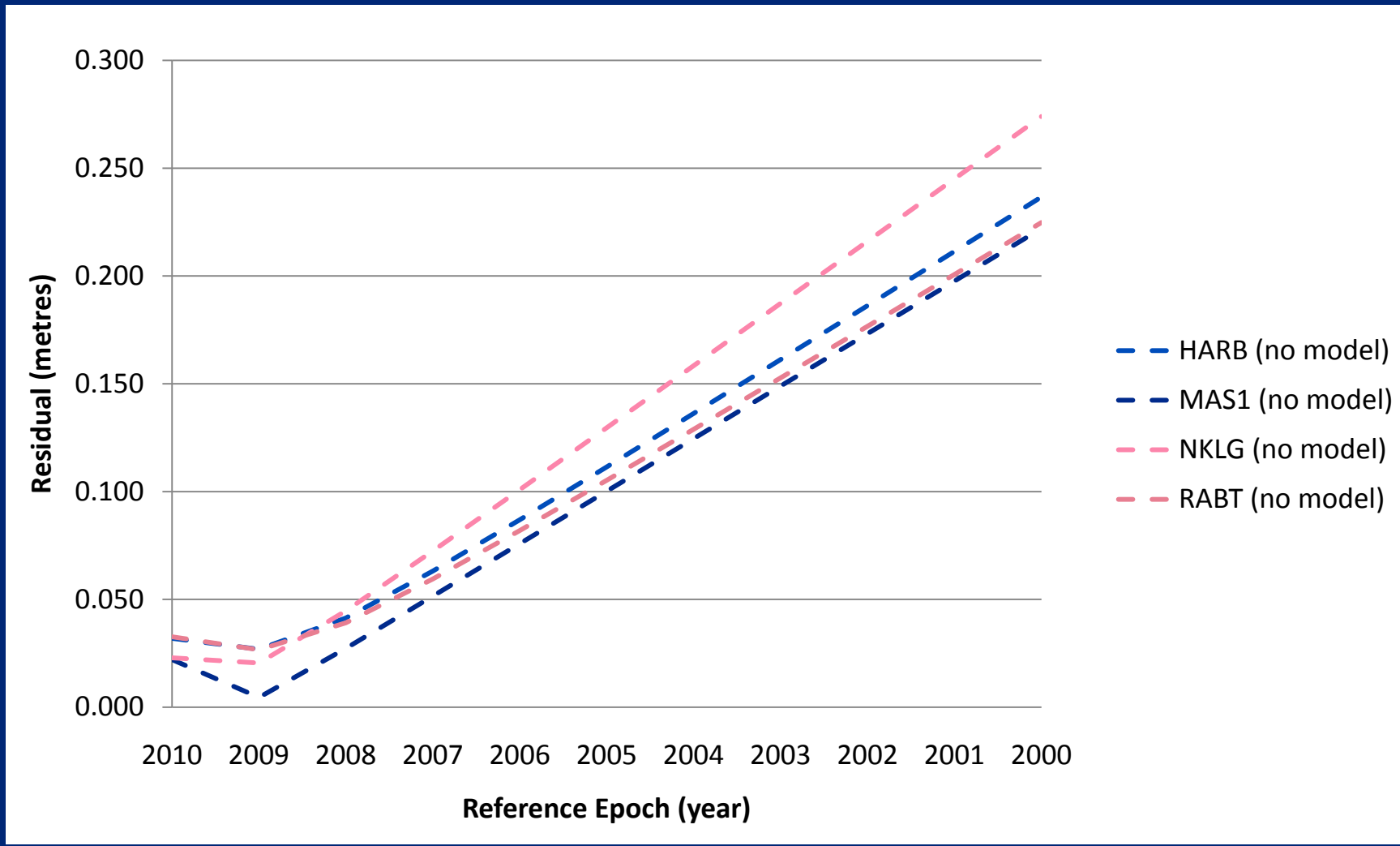
Principal Plates and Plate Boundaries in Africa

Stability of the Nubian Plate

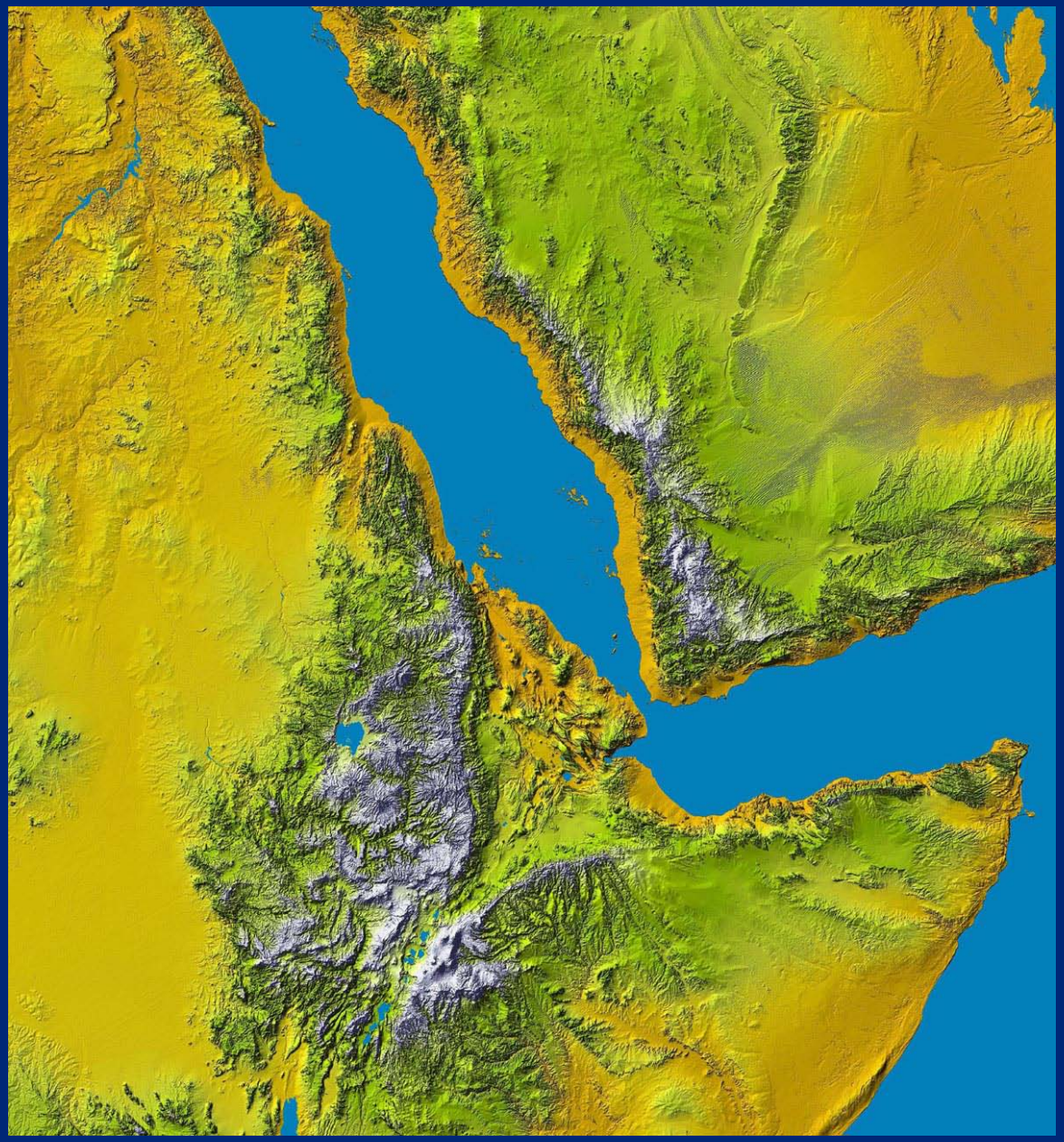


ITRF Site velocity

Deformation rates computed from ITRF2008 GPS SSC Solution
http://itrf.ensg.ign.fr

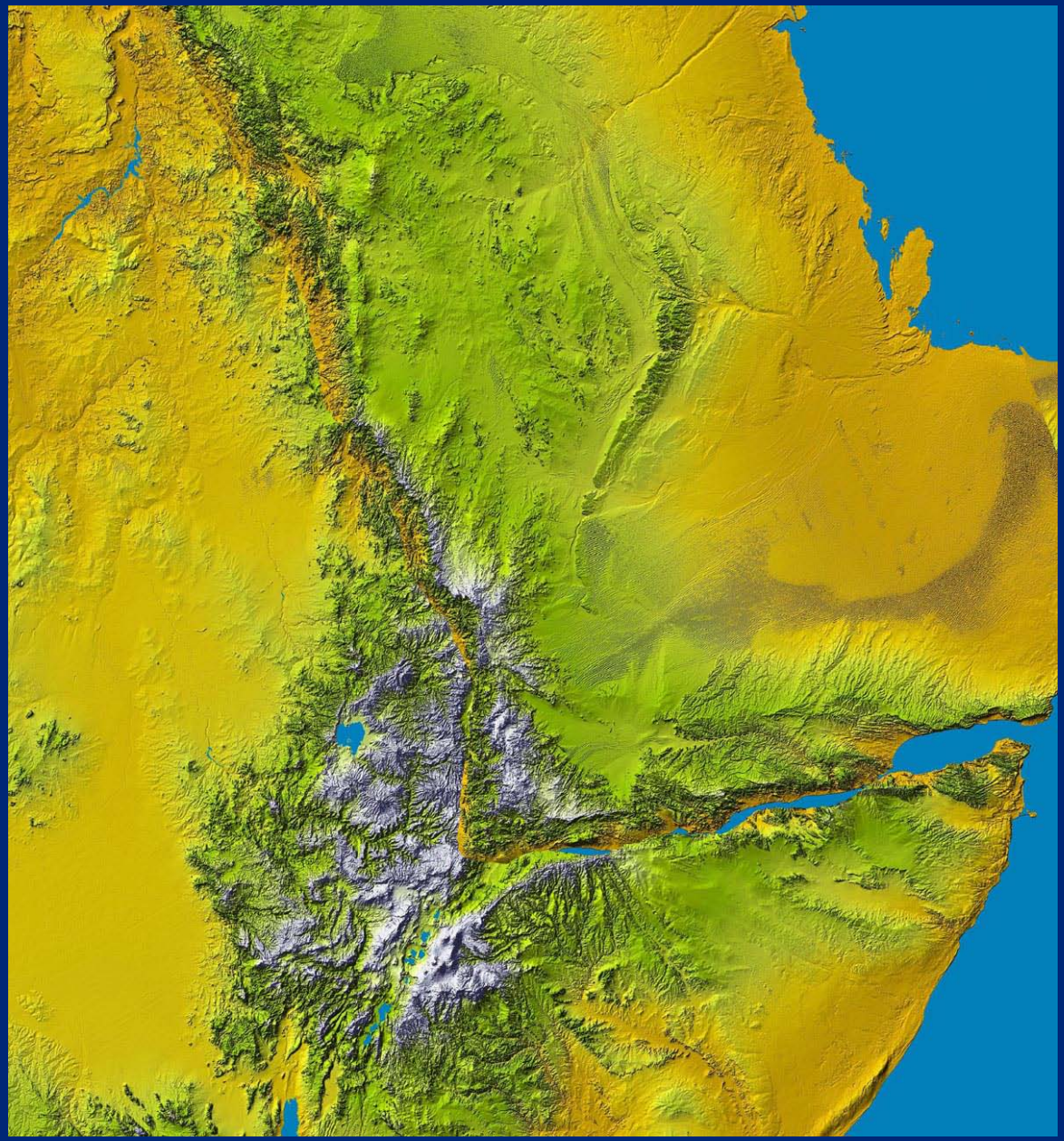


Divergence between ITRF (Epoch 2009) and ITRF (epoch of measurement)



Nubian and Arabian Plate Boundary today

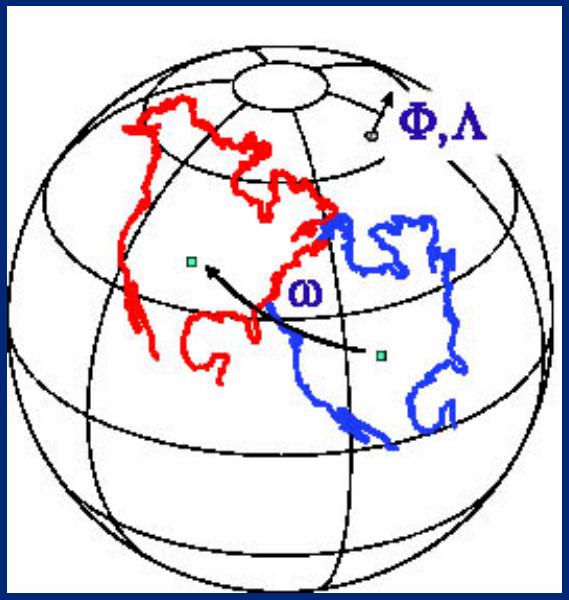
image:
JPL NASA SRTM



Nubian and Arabian Plate Boundary 30 Ma

$\Omega_X = \mathrm{COS}(\Phi)\mathrm{COS}(\Lambda)\omega$

Rigid Plate Model



Euler Poles to Cartesian rotation rates

$$\Omega_X = \mathrm{COS}(\Phi)\mathrm{COS}(\Lambda)\omega$$
$$\Omega_Y = \mathrm{COS}(\Phi)\mathrm{SIN}(\Lambda)\omega$$
$$\Omega_Z = \mathrm{SIN}(\Phi)\omega$$

ITRF2005 African plate parameters (Altamimi *et al.* 2007)

Plate	Euler pole of rotation			Equivalent Cartesian angular velocity		
	Φ (°)	Λ (°)	ω (°/Ma)	Ω_X (Rad/Ma)	Ω_Y (Rad/Ma)	Ω_Z (Rad/Ma)
Arabia	49.6	5.1	0.579	0.006518	0.000577	0.007700
Eurasia	56.3	-96.0	0.261	-0.000263	-0.002512	0.003791
Nubia	50.0	-82.5	0.269	0.000394	-0.002995	0.003594
Somalia	53.7	-89.5	0.309	0.000026	-0.003196	0.004344

Kinematic to Static transformation

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + S \cdot \left(\begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix} + \begin{bmatrix} \Omega_Y Z_t - \Omega_Z Y_t \\ \Omega_Z X_t - \Omega_X Z_t \\ \Omega_X Y_t - \Omega_Y X_t \end{bmatrix} \cdot (t_0 - t) \cdot 1\text{E-}6 \right)$$

"Static" coordinates at reference epoch

Local frame translation & scale (only if required)

"Measured" ITRF coordinates

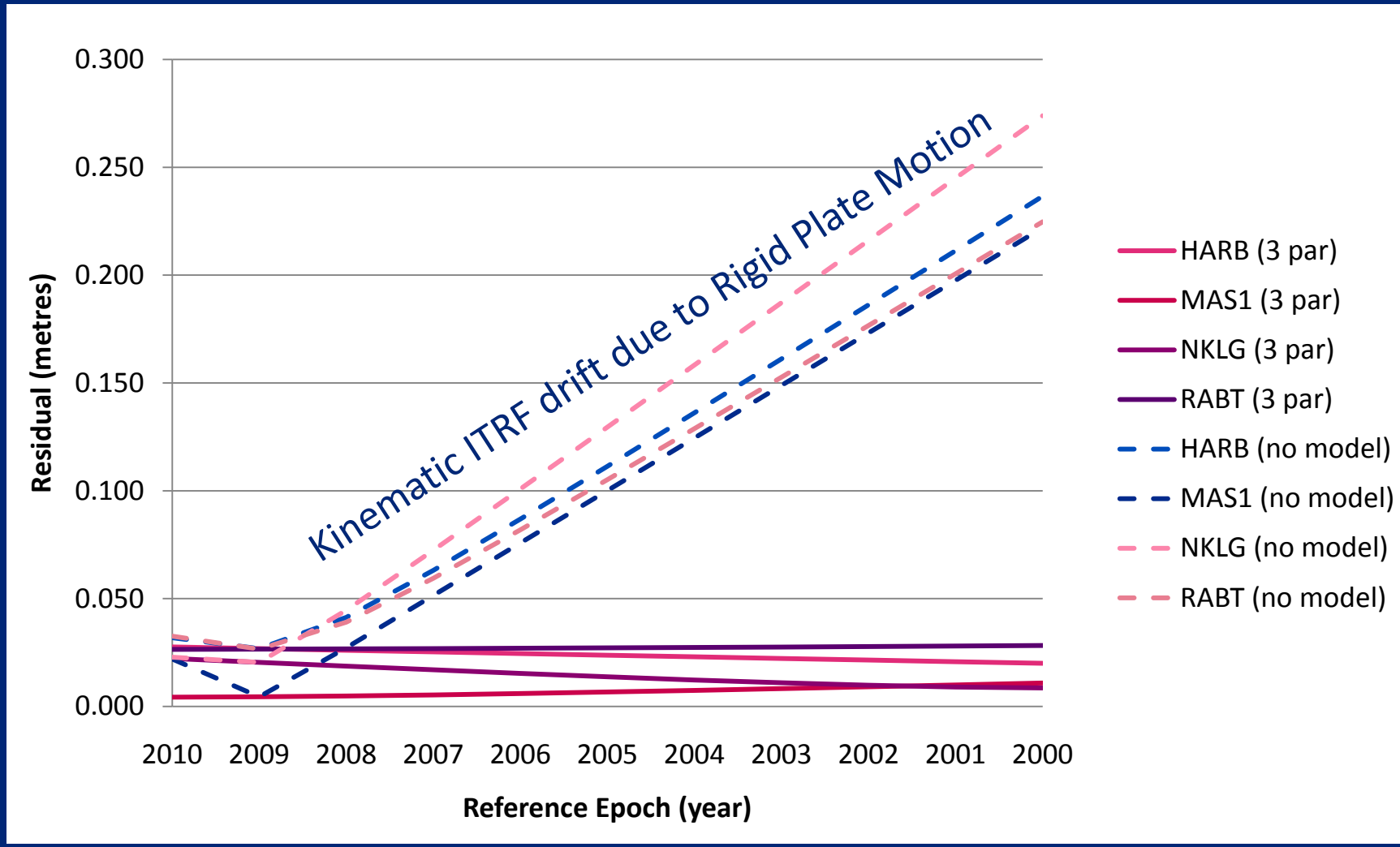
Plate rotation parameters

reference epoch

measurement epoch

$$X_0 = X_t + (\Omega_Y Z_t - \Omega_Z Y_t).(t_0 - t) \cdot 1\text{E-}6$$
$$Y_0 = Y_t + (\Omega_Z X_t - \Omega_X Z_t).(t_0 - t) \cdot 1\text{E-}6$$
$$Z_0 = Z_t + (\Omega_X Y_t - \Omega_Y X_t).(t_0 - t) \cdot 1\text{E-}6$$

Simplified 3-parameter equations Kinematic ITRF to Static ITRF
(no scale or translation parameters)



Improved coordinate consistency using a 3 parameter rigid plate transformation

Stanaway & Roberts, FIG Working Week, Marrakech, Morocco, 18-22 May 2011

Limitations of a Rigid Plate Model

Intraplate deformation not accounted for
(usually small magnitude < 1 mm/yr anyway)

Fails near plate boundaries
(requires additional modelling of locked faults)

Coseismic and Postseismic deformation not modelled

Thank you

Merci